# Mapping and Correcting the Wavefront of GQuEST End Mirrors

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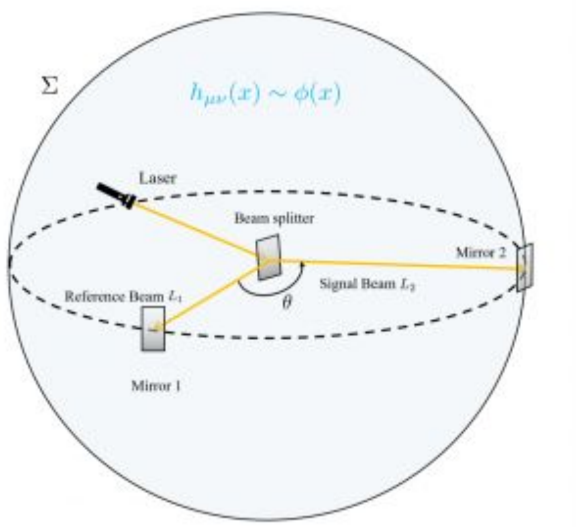
DCC: LIGO-G2401688

# Agenda

- What is GQuEST?
- Mirrors and modes
- Motivation
- Methods
- Wavefront measurement
- Results
  - Simulation
  - Shack-Hartmann
  - Fizeau Interferometer
- What’s next?

# Gravity from the **Qu**antum **E**ntanglement of **S**pace**T**ime (**GQuEST**)



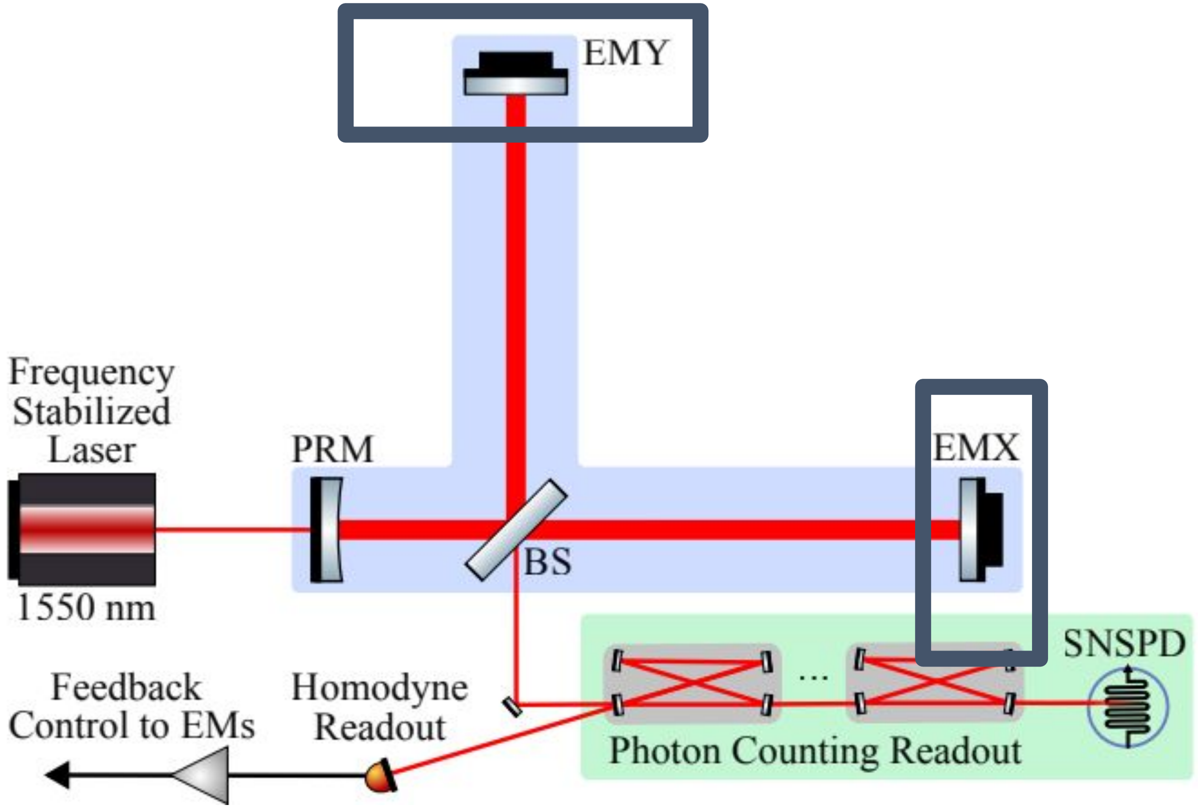

*Sphere over which fluctuations can be measured*

Goal: Sensitivity beyond the standard quantum limit by counting individual photons to measure extremely small fluctuations in space to test proposed theories of quantum gravity.

arXiv:2404.07524

# GQuEST

*Simplified GQuEST Setup*

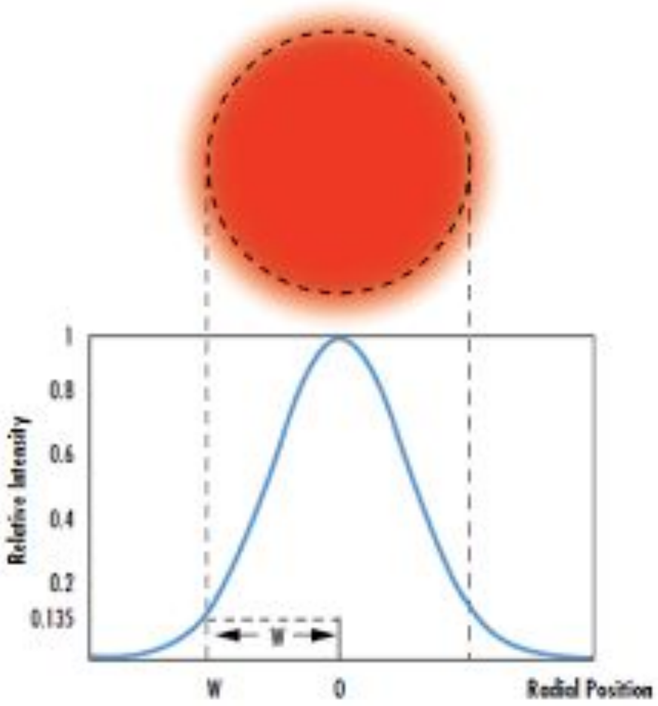



*Vacuum chamber used to hold mirror*

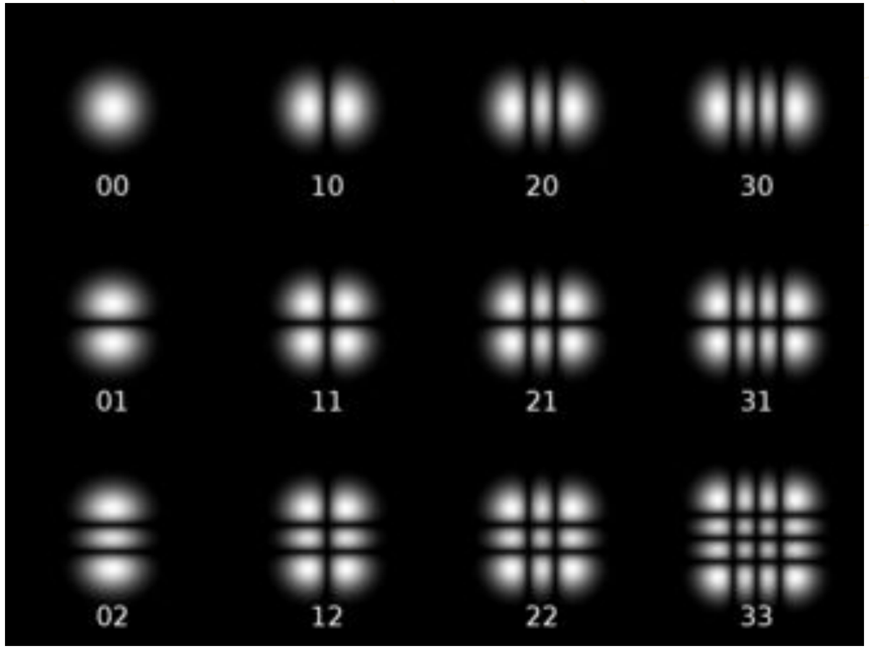
arXiv:2404.07524

# Mirrors and Modes

*Gaussian beam*

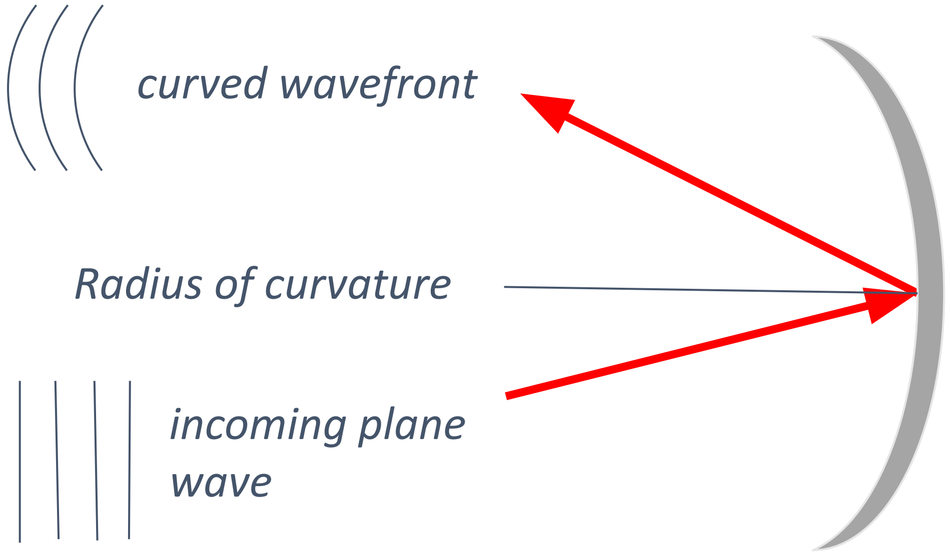



*12 Hermite Gauss modes*




*Hermite-Gauss beams: orthogonal family of laser modes*

# Radius and Coupling




*Coupling Coefficient:*

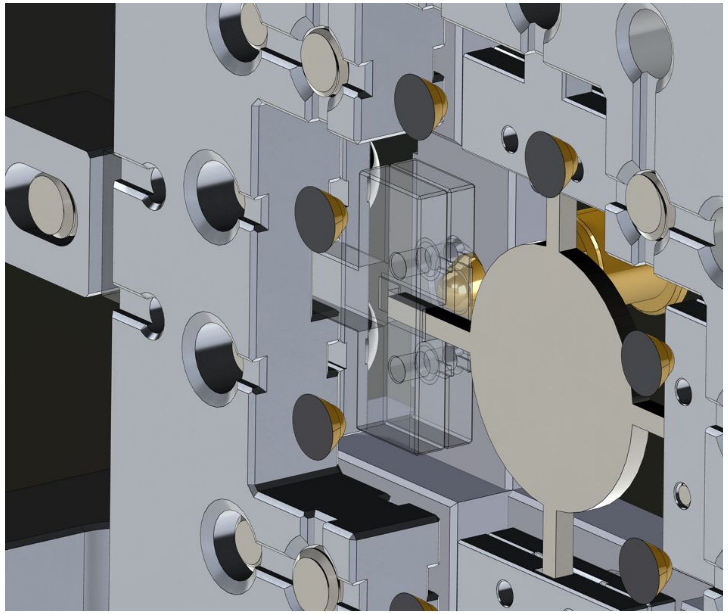
**How much of our incoming wave is contained in our outgoing wave?**

$$k_{mnm'n'}(x,y) = \int HG^*_{n'm'}(x,y) \exp\left[ik\phi\right] HG_{nm}(x,y)\, dx\, dy$$

Incoming Gaussian: $HG_{nm}(x,y)$

Outgoing Gaussian: $HG^*_{n'm'}(x,y)$

# Lensing into a mode



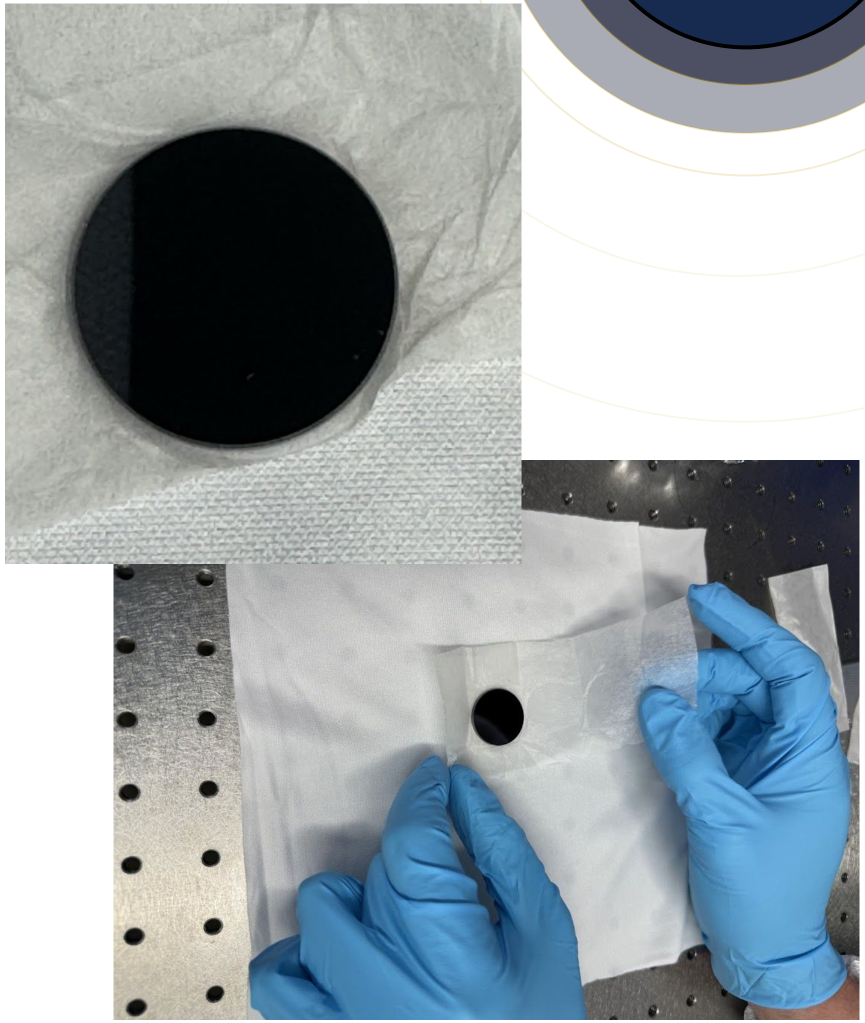
*Design of deformable end mirror*

- Two mirrors in the interferometer need their **modes** to match
- When they do not match, higher order modes create **contrast defects** (extra light in the interferometer)
- Curving the mirror ‘**lenses**’ the mirror into a mode we choose, and we can describe how much by coupling coefficients

# Motivation

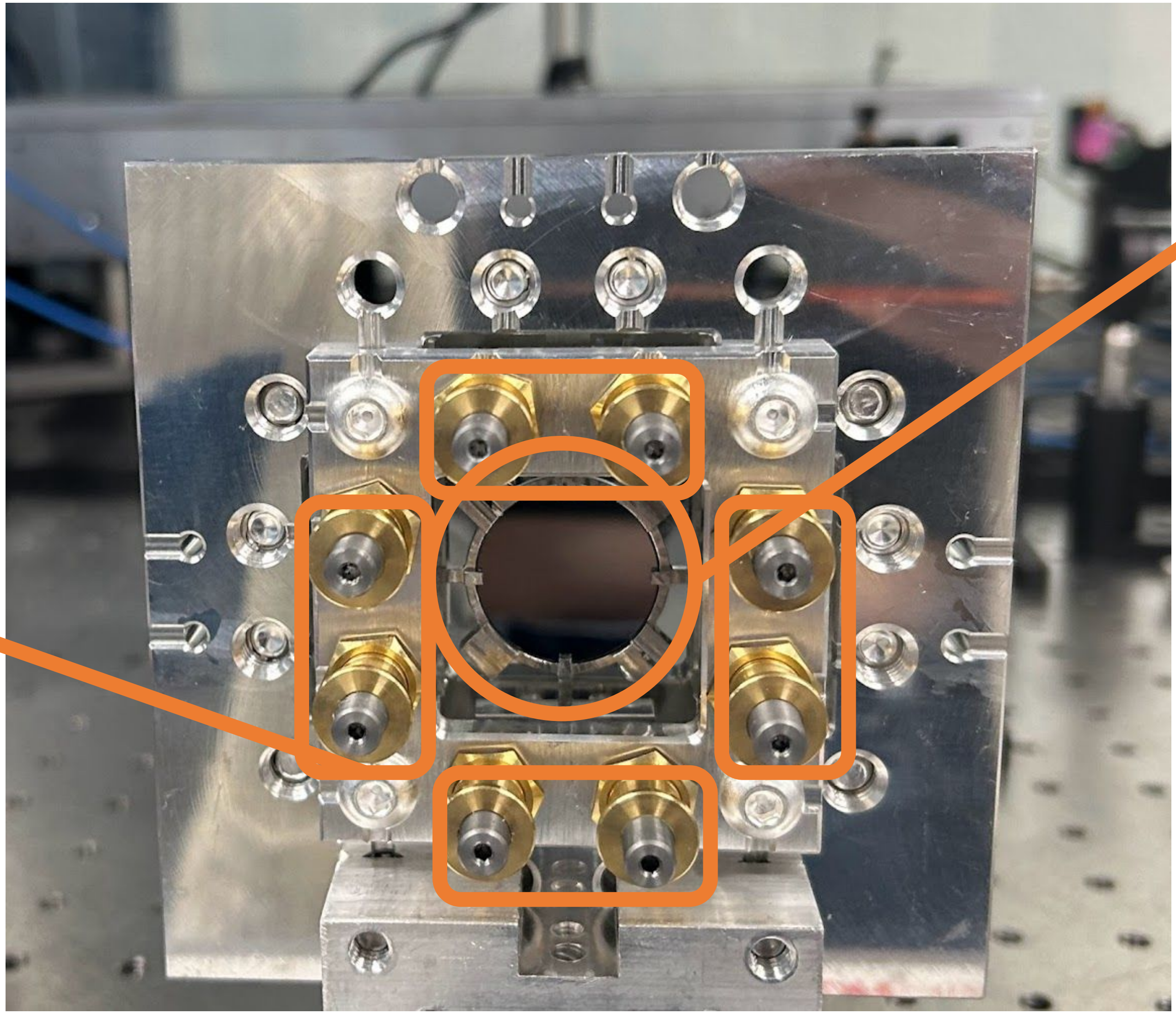
- GQuEST requires very thin mirrors to reduce mechanical noise
- These mirrors will be coated for high reflectivity, which adds stress
- Stress changes the curvature of the mirror
- Curvature changes modes of outgoing light
- **If the modes of the two mirrors do not match**, we add contrast defects
- Contrast defects affect measurement

This means we need wavefront correction!



*Mirrors, currently uncoated*

# Methods



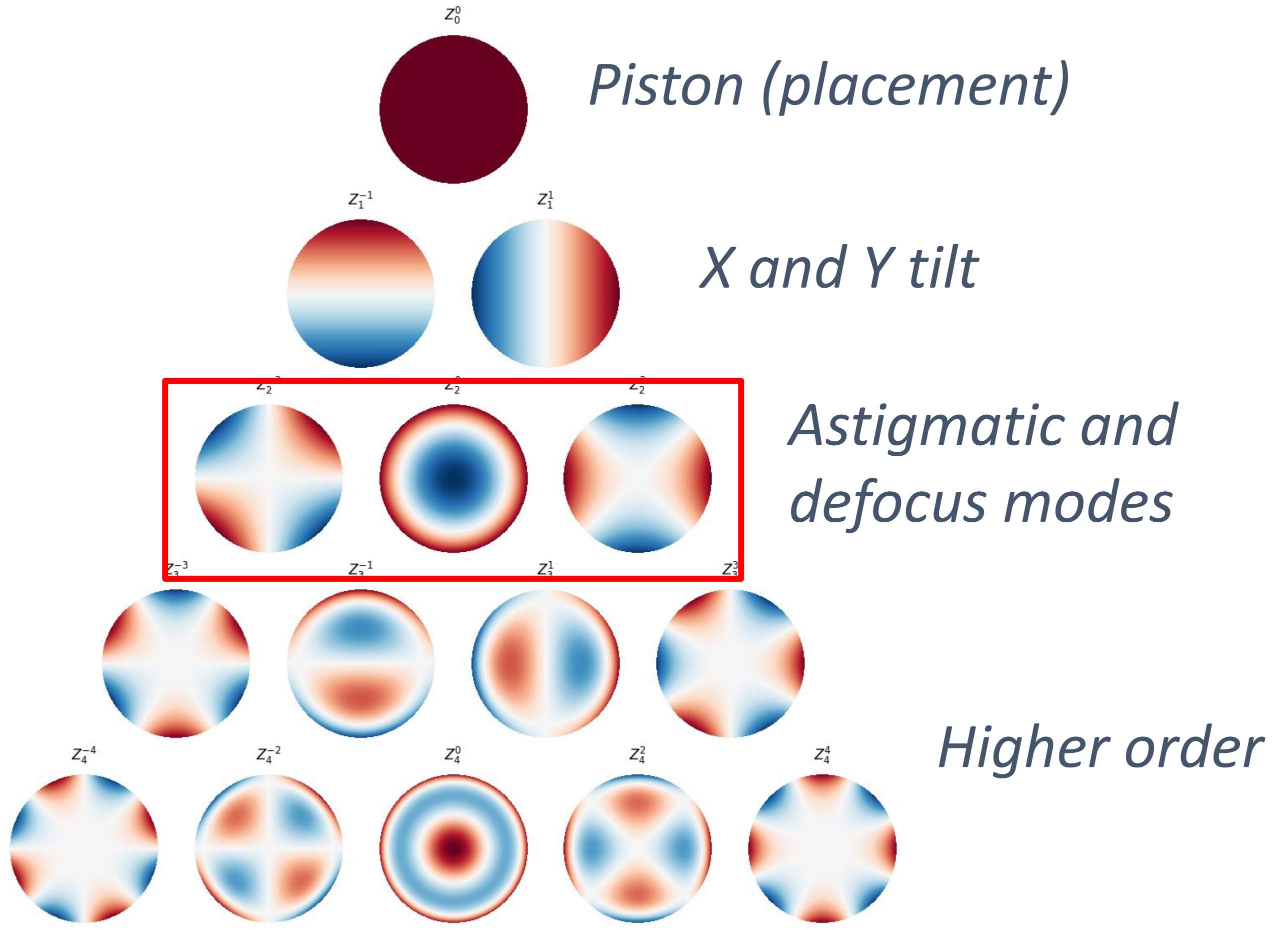
8 Adjustment screws
+4 in front

Silicon mirror

Half ring

End Mirror Mount

# Lensing into higher order modes




*Zernike Polynomials, orthogonal spherical special functions*

- Mirror is physically curved into higher order modes by screws
- Modes above are loss (contribute to contrast defects)
- Modes below can be 'zeroed' by placement of the mirror

# Lensing into higher order modes

*Zernike Polynomials, orthogonal spherical special functions*

*Oblique astigmatism (order 3)*

*Defocus mode (order 4)*

*Vertical astigmatism (order 5)*

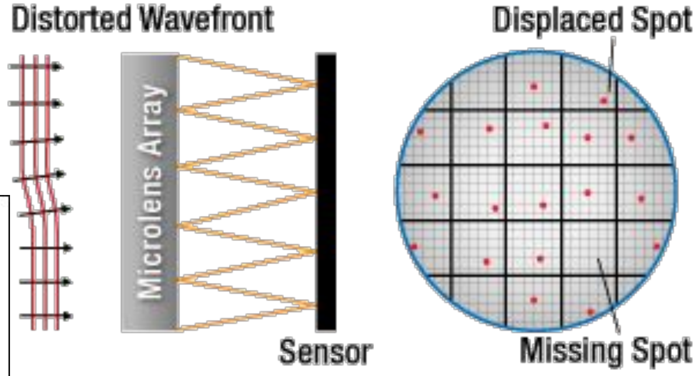
# Wavefront Measurement Methods

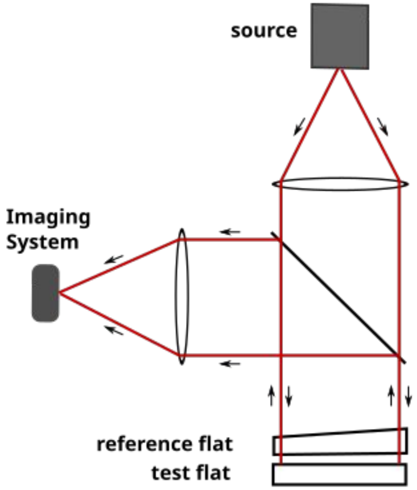
Shack-Hartmann Wavefront Sensor:

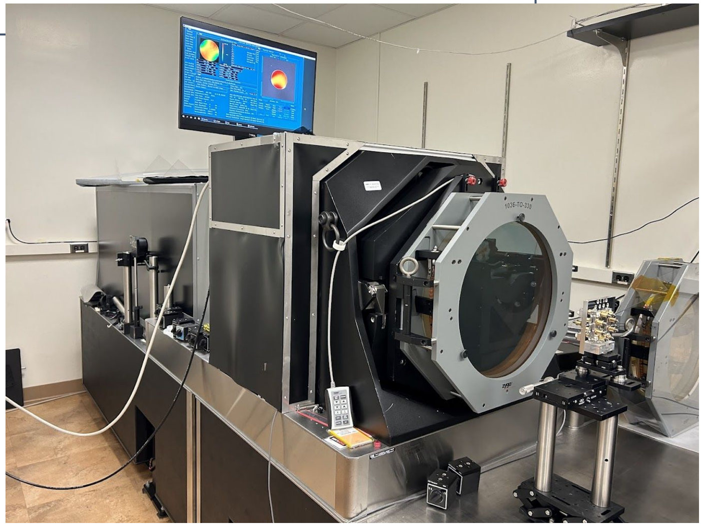
- Less accurate of the two
- Uses a differential measurement from an array of powerful microlenses to reconstruct the wavefront

Fizeau Interferometer:

- More accurate, used for LIGO Test Masses
- Measures difference in interference fringes between reference and test flat to reconstruct wavefront

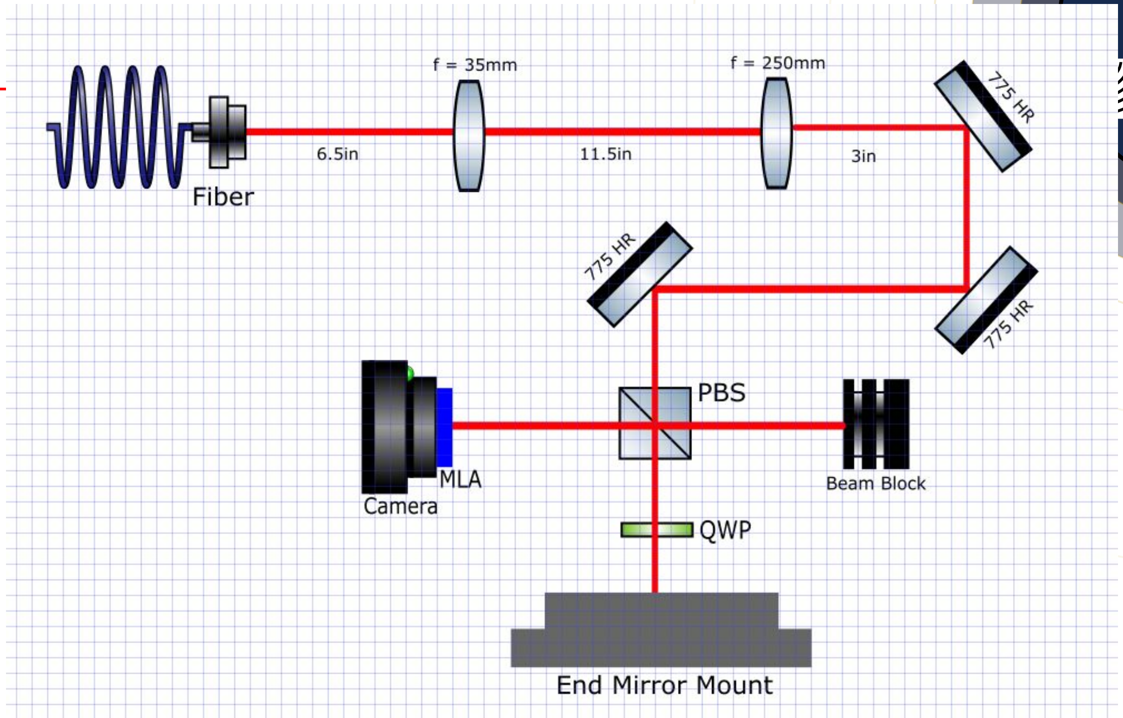



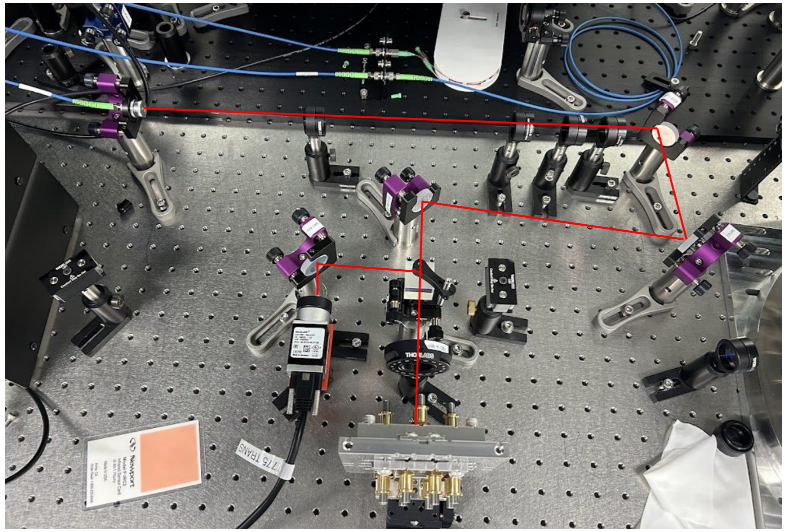



# Methods

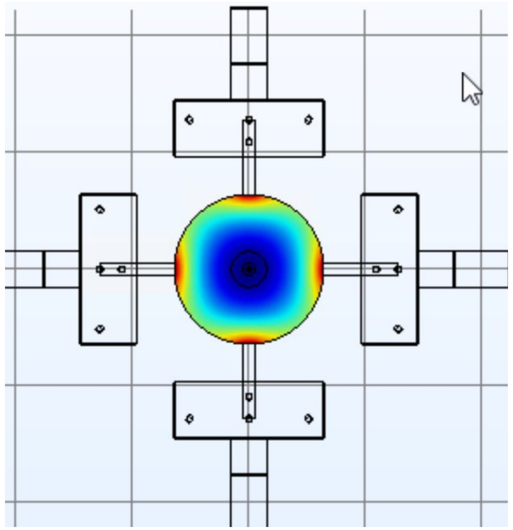
- Imaged mirror without mount with the Fizeau
- Developed a pipeline for analyzing wavefront data, and using it to determine radius of curvature and displacement of the mirror
- Designed and built a setup for using the Shack-Hartmann Wavefront Sensor, took data with different modes
- Imaged different modes with the Fizeau interferometer



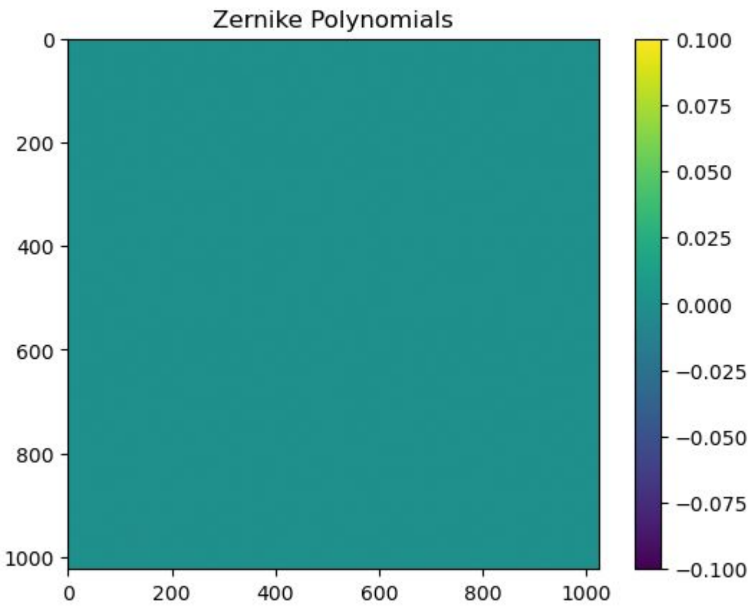



*Setup design for Shack-Hartmann Sensor*

# Simulation

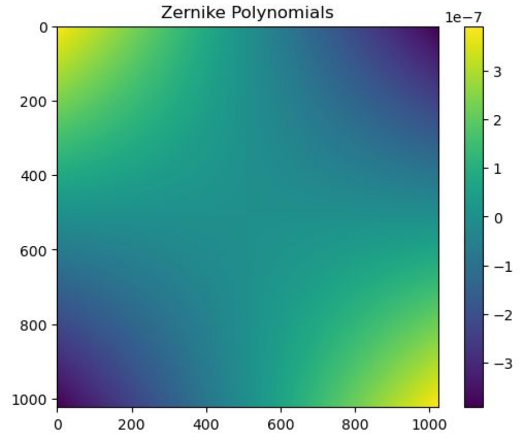


*COMSOL simulation of adding pressure to a mirror*

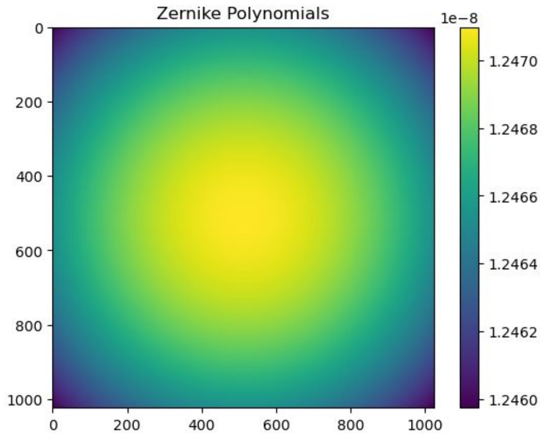



| Mode | Coupling | Radius (D) |
|---|---|---|
| HG00 | 0.99997 | 4.75E-15 |
| HG01 | 1.86E-17 | - |
| HG10 | 1.39E-17 | - |
| HG02 | 1.97E-16 | 1.29E-17 |
| HG11 | 5.63E-19 | 4.37E-20 |
| HG20 | 1.66E-16 | 1.53E-17 |

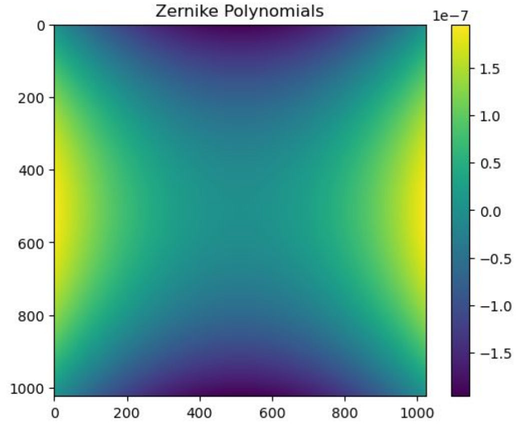
# Simulation




| Mode | Coupling | Radius (D) |
|---|---|---|
| HG00 | 0.99949541 | 5.54E-05 |
| HG01 | 2.63E-17 | - |
| HG10 | 1.37E-17 | - |
| HG02 | 0.00071305 | 5.53E-05 |
| HG11 | 0.03173144 | 2.46E-03 |
| HG20 | 0.00071305 | 5.53E-05 |



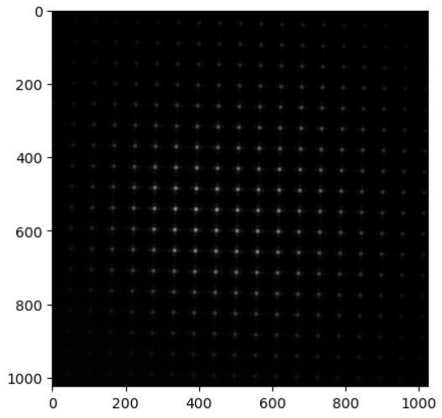

| Mode | Coupling | Radius (D) |
|---|---|---|
| HG00 | 0.99999999 | 5.02E-14 |
| HG01 | 2.81E-17 | - |
| HG10 | 1.05E-17 | - |
| HG02 | 6.43E-07 | 4.99E-08 |
| HG11 | 2.80E-18 | 2.17E-19 |
| HG20 | 6.43E-07 | 4.99E-08 |

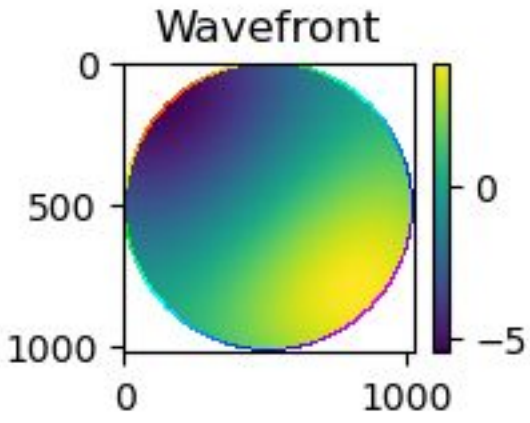


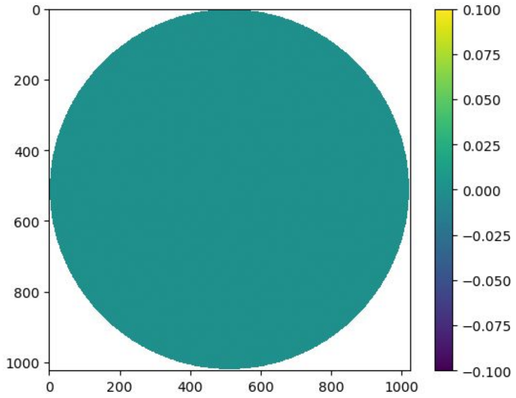

| Mode | Coupling | Radius (D) |
|---|---|---|
| HG00 | 0.9994954 | 5.54E-05 |
| HG01 | 1.15E-16 | - |
| HG10 | 1.37E-17 | - |
| HG02 | 0.022448 | 1.74E-03 |
| HG11 | 1.37E-18 | 1.06E-19 |
| HG20 | 0.022448 | 1.74E-03 |

# Shack-Hartmann Sensor

*Flat wavefront*

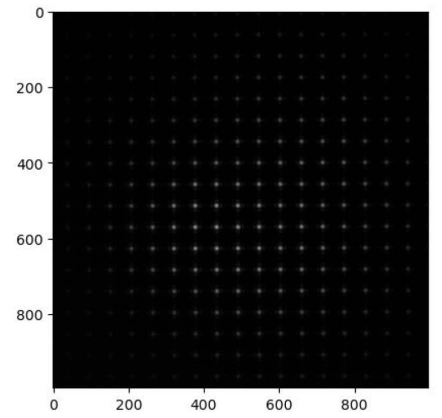


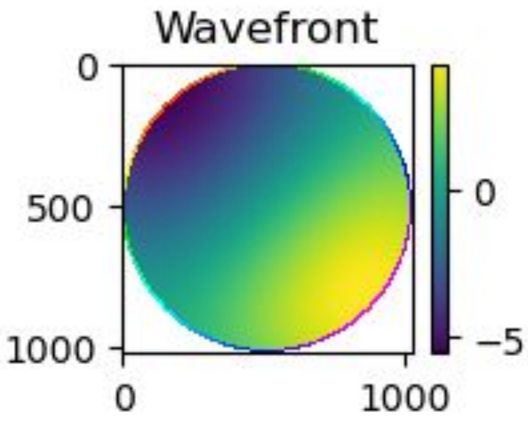




*Subtracted wavefronts*

*Curved wavefront*



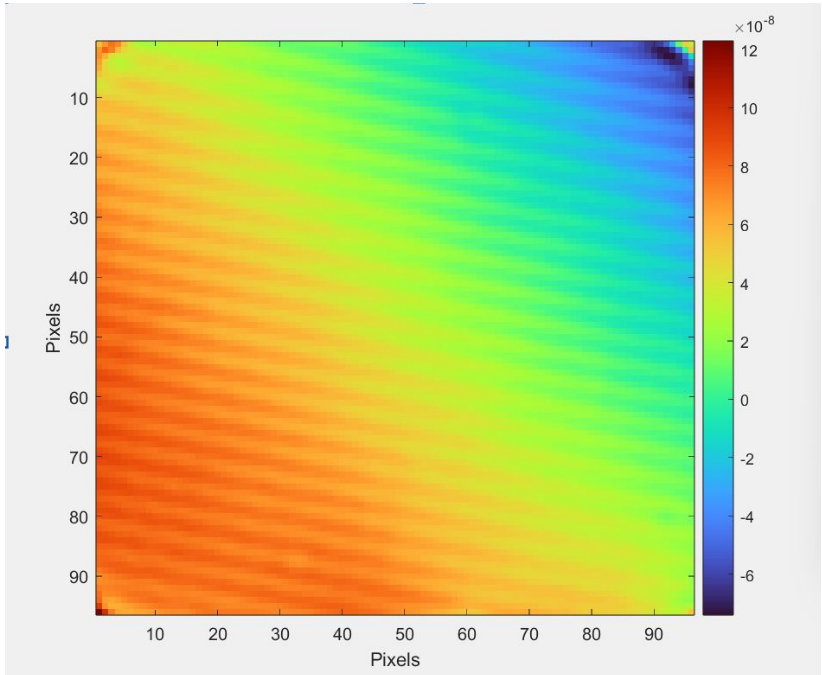

***Shack - Hartmann***

*Data from microlens array*

*Reconstructed wavefront*

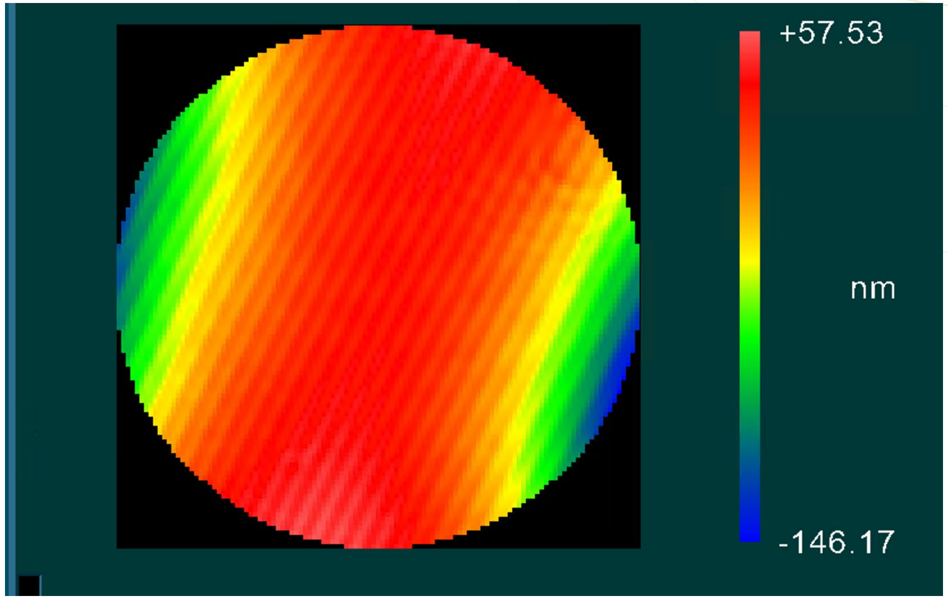
# Fizeau: Flat mirror

*Mirror outside of mount*




| | |
|---|---|
| Coupling | 0.999349524 |
| Radius (D) | 7.13878698e-05 |

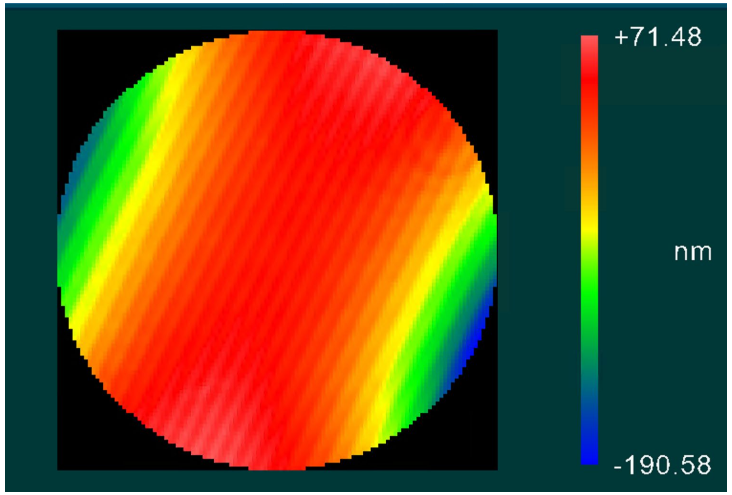
*Mirror inside mount*



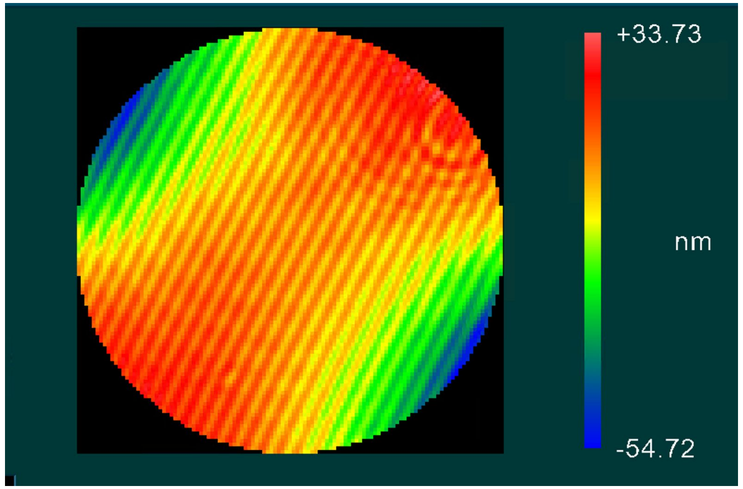

| | |
|---|---|
| Coupling | 0.999553816 |
| Radius (D) | 4.895232743e-05 |

# Fizeau continued: curved mirrors

Oblique astigmatic mode ('+ shape')




*Wavefront image*

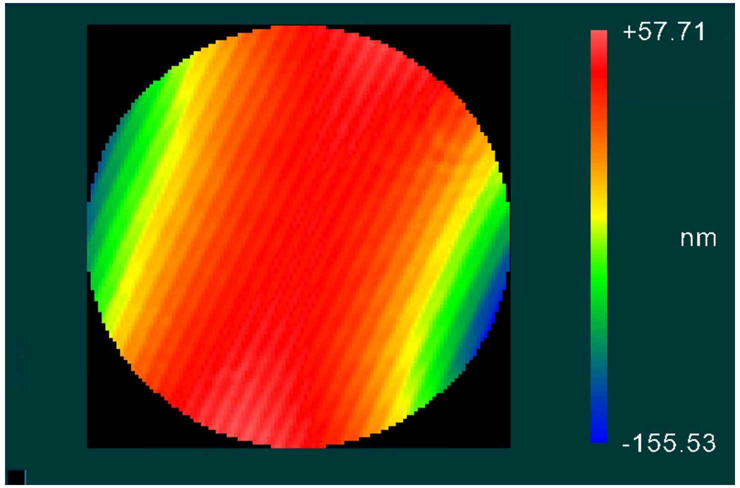



*Net change from unstressed mirror*

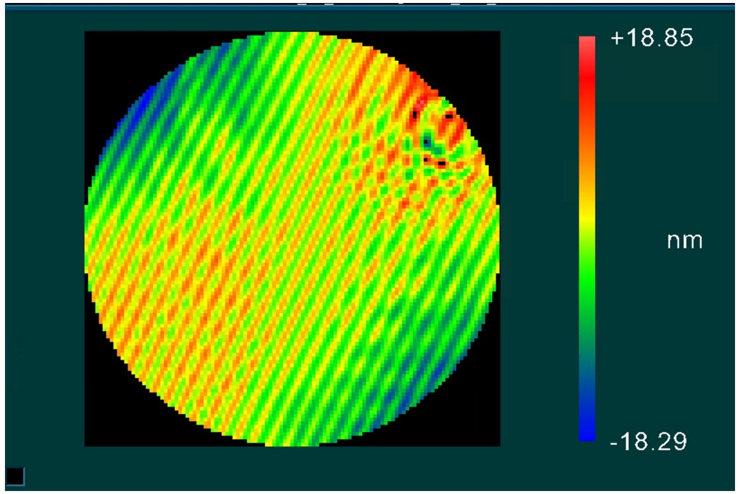
| Plus | Coefficient | | Radius (D) | |
|---|---|---|---|---|
| Mode | Simulated | Actual | Simulated | Actual |
| HG00 | 0.9994954 | 0.9999701 | 5.54E-05 | 3.27E-06 |
| HG01 | 2.63E-17 | 1.20E-05 | - | - |
| HG10 | 1.37E-17 | 1.02E-04 | - | - |
| HG02 | 0.00071305 | 0.000123028 | 5.53E-05 | 1.59E-05 |
| HG11 | 0.0317314 | 0.0010359 | 2.46E-03 | 8.03E-05 |
| HG20 | 0.0007130 | 0.0002057 | 5.53E-05 | 9.54E-06 |

# Fizeau continued: curved mirrors

## Defocus mode ('o shape')




*Wavefront image*

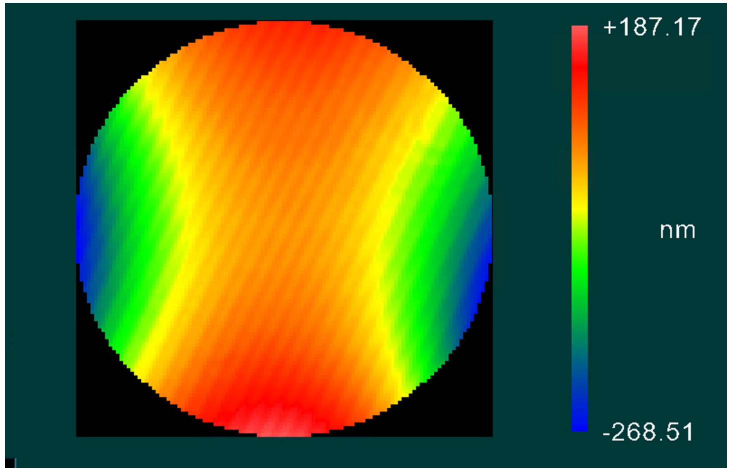



*Net change from unstressed mirror*

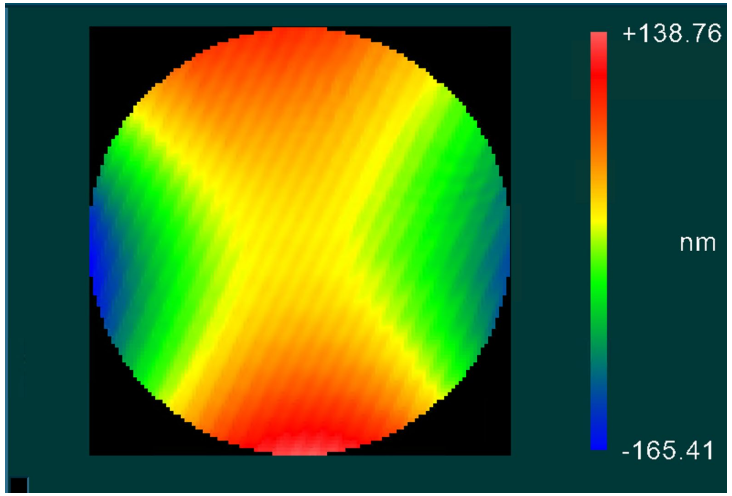
| o | Coefficient | | Radius (D) | |
|---|---|---|---|---|
| mode | Simulated | Actual | Simulated | Actual |
| HG00 | 0.999999 | 0.999928 | 5.02E-14 | 7.85E-06 |
| HG01 | 2.81E-17 | 7.83E-05 | - | - |
| HG10 | 1.05E-17 | 3.28E-05 | - | - |
| HG02 | 6.43E-07 | 5.36E-04 | 4.99E-08 | 3.07E-05 |
| HG11 | 2.80E-18 | 1.42E-04 | 2.17E-19 | 1.10E-05 |
| HG20 | 6.43E-07 | 3.96E-04 | 4.99E-08 | 4.15E-05 |

# Fizeau continued: curved mirrors

Vertical astigmatic mode ('x shape')



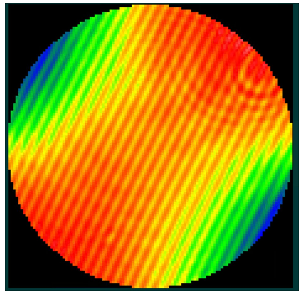

*Wavefront image*



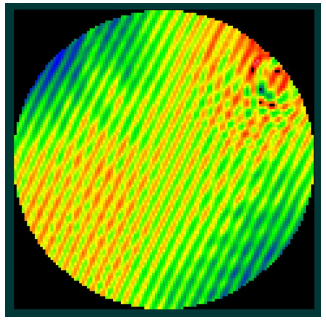

*Net change from unstressed mirror*

| x | Coefficient | | Radius (D) | |
|---|---|---|---|---|
| mode | Simulated | Actual | Simulated | Actual |
| HG00 | 0.9994954 | 0.9999837 | 5.54E-05 | 1.78E-06 |
| HG01 | 1.15E-16 | 7.60E-05 | - | - |
| HG10 | 1.37E-17 | 6.20E-05 | - | - |
| HG02 | 0.0224488 | 0.0021739 | 1.74E-03 | 2.23E-04 |
| HG11 | 1.37E-18 | 3.44E-04 | 1.06E-19 | 2.67E-05 |
| HG20 | 0.0224488 | 0.0028726 | 1.74E-03 | 1.69E-04 |

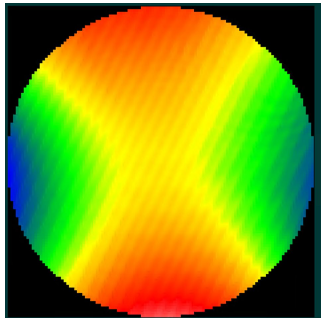
# Fizeau continued: curved mirrors




*Oblique astigmatism*
*(order 3)*




*Defocus mode*
*(order 4)*




*Vertical astigmatism*
*(order 5)*

# Application to GQuEST: Loss

| | Simulated | Fizeau | Loss (ppm) |
|---|---|---|---|
| Flat mirror | 8.66E-14 | 5.17E-05 | 51.7 |
| Plus | 1.02E-06 | 2.51E-05 | 25.1 |
| O | 8.82E-14 | 4.11E-05 | 41.1 |
| X | 1.02E-06 | 5.11E-05 | 51.1 |

- Loss is a measure of how many photons are scattered into higher order modes
- GQuEST has a loss budget on the order of ~100 ppm at most for mirrors
- Important to make sure mirrors are not a major contributor to this overall loss

# Conclusions

- We can properly lens into the astigmatic modes, but have difficulty doing so into the defocus mode
- The Shack-Hartmann Sensor, while helpful, is not precise enough to measure changes in modes
- The modes produced by the end mirror mount generally matches the COMSOL and python simulations
- Positive first step towards making mirrors that match in modes

# Next Steps

- Implement a different data analysis process for the Shack-Hartmann sensor to potentially improve its ability to reconstruct wavefronts
- Continue to use and improve the pipeline for analyzing data
- Continue trying to lens into the defocus mode
- Reducing loss into higher order modes
- Replacing these mirrors in the half ring with spokes and mirrors as one piece
- Coating the mirrors

# Questions?

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# Extra Info and Slides

# How did we make the Shack-Hartmann wavefront?

$$\Phi(x,y) = \sum_{i=1}^{I} a_i Z_i(x,y)$$

$$\frac{\partial \Phi}{\partial x} = \sum_{i=1}^{I} a_i \frac{\partial Z_i(x,y)}{\partial x}$$

$$\frac{\partial \Phi}{\partial y} = \sum_{i=1}^{I} a_i \frac{\partial Z_i(x,y)}{\partial y}$$

$$P = (\text{x shift 1, x shift 2, .... x shift } k\text{, y shift 1, y shift 2....y shift } k)$$

$$D = \begin{pmatrix} \frac{\partial Z_2(x,y)_1}{\partial x} & \frac{\partial Z_2(x,y)_2}{\partial x} & \cdots & \frac{\partial Z_2(x,y)_1}{\partial y} & \cdots & \frac{\partial Z_2(x,y)_k}{\partial y} \\ \cdots & \cdots. & & \cdots & & \cdots \\ \frac{\partial Z_i(x,y)_1}{\partial x} & \frac{\partial Z_i(x,y)_2}{\partial x} & \cdots & \frac{\partial Z_i(x,y)_1}{\partial y} & \cdots & \frac{\partial Z_i(x,y)_k}{\partial y} \end{pmatrix}$$

$$P = D^t A$$

$$DP = DD^t A$$

$$A = (DD^t)^{-1} DP$$

$$A = (a_1, a_2, a_3 \ldots . a_i)$$

# How does coupling relate to radius of curvature?

$$u_n(x,z) = \left(\frac{2}{\pi}\right)^{1/4} \left(\frac{1}{2^n n! w_0}\right) \left(\frac{q_0}{q(z)}\right)^{1/2} \left(\frac{q_0 q^*(z)}{q_0^* q(z)}\right)^{n/2} H_n\left(\frac{\sqrt{2}x}{w(z)}\right) \exp\left(-i\frac{kx^2}{2q(z)}\right)$$

$$k_{mn}(D, q_1, q_2) = \int u_n^*(x, q_1) e^{i\frac{kDx^2}{2}} u_m(x, q_2)\, dx$$

$$k_{00} = (1 - \frac{i}{4} Dkw^2)^{-1/2}$$

$$k_{02} = \frac{1}{\sqrt{2}} \frac{\frac{1}{4} Dkw^2}{(1 - \frac{i}{4} Dkw^2)^{3/2}}$$

$$D = \frac{-4i}{kw^2}\left(1 - \frac{1}{k_{00}^2}\right)$$

$$D \approx \frac{4\sqrt{2}k_{02}}{kw^2}$$

# What are Zernike Polynomials?

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\cos(m\phi)$$

$$W(\rho,\phi) = \sum_{n,m} a_{nm} Z_n^m(\rho,\phi)$$

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\sin(m\phi)$$

$$j = \frac{n(n+1)+m}{2}$$

$$R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n+1}{2}-k)!(\frac{n-m}{2}-k)!}\rho^{n-2k}$$

# What do the adjustment screws do?